

**Search for the Decay $K^+ \rightarrow \pi^+ \gamma \gamma$
in the π^+ Momentum Region $P > 213 \text{ MeV}/c$**

E949 Collaboration

A.V. Artamonov^a, B. Bassalleck^b, B. Bhuyan^{c,1},
E.W. Blackmore^d, D.A. Bryman^e, S. Chen^{d,2}, I-H. Chiang^c,
I.-A. Christidi^f, P.S. Cooper^g, M.V. Diwan^c, J.S. Frank^c,
T. Fujiwara^h, J. Hu^d, D.E. Jaffe^c, S. Kabeⁱ, S.H. Kettell^c,
M.M. Khabibullin^j, A.N. Khotjantsev^j, P. Kitching^k,
M. Kobayashiⁱ, T.K. Komatsubaraⁱ, A. Konaka^d,
A.P. Kozhevnikov^a, Yu.G. Kudenko^j, A. Kushnirenko^{g,3},
L.G. Landsberg^a, B. Lewis^b, K.K. Li^c, L.S. Littenberg^c,
J.A. Macdonald^{d,4}, J. Mildemberger^d, O.V. Mineev^j,
M. Miyajima^l, K. Mizouchi^h, V.A. Mukhin^a, N. Muramatsu^m,
T. Nakano^m, M. Nomachiⁿ, T. Nomura^h, T. Numao^d,
V.F. Obraztsov^a, K. Omataⁱ, D.I. Patalakha^a, S.V. Petrenko^a,
R. Poutissou^d, E.J. Ramberg^g, G. Redlinger^c, T. Satoⁱ,
T. Sekiguchiⁱ, T. Shinkawa^o, R.C. Strand^c, S. Sugimotoⁱ,
Y. Tamagawa^l, R. Tschirhart^g, T. Tsunemi^{i,5}, D.V. Vavilov^a,
B. Viren^c, N.V. Yershov^j, Y. Yoshimuraⁱ, T. Yoshioka^{i,6}

^a*Institute for High Energy Physics, Protvino, Moscow Region, 142 280, Russia*

^b*Department of Physics and Astronomy, University of New Mexico, Albuquerque,
NM 87131, USA*

^c*Brookhaven National Laboratory, Upton, NY 11973, USA*

^d*TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3*

^e*Department of Physics and Astronomy, University of British Columbia,
Vancouver, British Columbia, Canada V6T 1Z1*

^f*Department of Physics and Astronomy, Stony Brook University, Stony Brook, NY
11794, USA*

^g*Fermi National Accelerator Laboratory, Batavia, IL 60510, USA*

^h*Department of Physics, Kyoto University, Sakyo-ku, Kyoto 606-8502, Japan*

ⁱ*High Energy Accelerator Research Organization (KEK), Oho, Tsukuba, Ibaraki
305-0801, Japan*

^j*Institute for Nuclear Research RAS, 60 October Revolution Pr. 7a, 117312
Moscow, Russia*

^k*Centre for Subatomic Research, University of Alberta, Edmonton, Canada T6G
2N5*

^l*Department of Applied Physics, Fukui University, 3-9-1 Bunkyo, Fukui, Fukui
910-8507, Japan*

^m*Research Center for Nuclear Physics, Osaka University, 10-1 Mihogaoka,
Ibaraki, Osaka 567-0047, Japan*

ⁿ*Laboratory of Nuclear Studies, Osaka University, 1-1 Machikaneyama, Toyonaka,
Osaka 560-0043, Japan*

^o*Department of Applied Physics, National Defense Academy, Yokosuka, Kanagawa
239-8686, Japan*

Abstract

We have searched for the $K^+ \rightarrow \pi^+ \gamma \gamma$ decay in the kinematic region with π^+ momentum close to the end point. No events were observed, and a 90% confidence-level upper limit on the partial branching ratio is obtained, $B(K^+ \rightarrow \pi^+ \gamma \gamma, P > 213 \text{ MeV}/c) < 8.3 \times 10^{-9}$ under the assumption of chiral perturbation theory including next-to-leading order “unitarity” corrections. The same data were used to determine an upper limit on the $K^+ \rightarrow \pi^+ \gamma$ branching ratio of 2.3×10^{-9} .

Key words: kaon rare decay, chiral perturbation theory, unitarity corrections, noncommutative theories

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¹ Also at the Department of Physics and Astrophysics, University of Delhi, Delhi 110007, India. Present address: Department of Physics and Astronomy, University of Victoria, Victoria, British Columbia, Canada V8W 3P6.

² Present address: Department of Engineering Physics, Tsinghua University, Beijing 100084, P.R. China.

³ Present address: Institute for High Energy Physics, Protvino, Russia.

⁴ Deceased.

⁵ Present address: Research Center for Nuclear Physics, Osaka University, Japan.

⁶ Present address: International Center for Elementary Particle Physics, University of Tokyo, Tokyo 113-0033, Japan.

We report the results of a search for the rare decay $K^+ \rightarrow \pi^+ \gamma \gamma$ in the π^+ momentum region $P > 213 \text{ MeV}/c$ from the E949 experiment [1] at the Alternating Gradient Synchrotron (AGS) of Brookhaven National Laboratory. The first observation of the decay in the π^+ momentum region 100–180 MeV/ c was reported [2] by the E787 experiment at the AGS with a partial branching ratio of $B(K^+ \rightarrow \pi^+ \gamma \gamma, 100 \text{ MeV}/c < P < 180 \text{ MeV}/c) = (6.0 \pm 1.5(\text{stat}) \pm 0.7(\text{syst})) \times 10^{-7}$. In the region $P > 215 \text{ MeV}/c$ no $K^+ \rightarrow \pi^+ \gamma \gamma$ decay was observed and, assuming phase-space kinematic distribution, a 90% confidence-level (C.L.) upper limit of 5.0×10^{-7} was set on the total branching ratio [2]. This established that the $K^+ \rightarrow \pi^+ \gamma \gamma$ background to the study of the rare decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ [3] in the E787 and E949 experiments using kaon decays at rest was negligible.

In an effective-field approach to low energy QCD called chiral perturbation theory (ChPT) [4], there is no tree-level $O(p^2)$ contribution to $K^+ \rightarrow \pi^+ \gamma \gamma$ or the neutral counterpart $K_L^0 \rightarrow \pi^0 \gamma \gamma$; the leading contributions start at $O(p^4)$ [5]. For $K^+ \rightarrow \pi^+ \gamma \gamma$, both the branching ratio and the π^+ spectrum shape are sensitive to the undetermined coupling-constant \hat{c} . There is no complete calculation at the next-to-leading order, $O(p^6)$; the dominant effects [6] are supposed to be 1) one-loop “unitarity” corrections, which are deduced from an empirical fit of the decay amplitude of $K^+ \rightarrow \pi^+ \pi^+ \pi^-$ and contain the same constant \hat{c} , and 2) the contribution of vector-meson exchange, which is expected to be negligible in $K^+ \rightarrow \pi^+ \gamma \gamma$ compared to unitarity corrections. The corrections predict the π^+ spectrum with a slightly different shape (Fig. 1). The measured π^+ spectrum of E787, Fig. 2 in [2], was consistent both with unitarity corrections ($\hat{c} = 1.8 \pm 0.6$) and without unitarity corrections ($\hat{c} = 1.6 \pm 0.6$); the E787 data preferred the inclusion of the corrections but

were not conclusive. For $K_L^0 \rightarrow \pi^0 \gamma \gamma$, the amplitude at $O(p^4)$ is determined unambiguously but the measured branching ratio, $(1.41 \pm 0.12) \times 10^{-6}$ [7], is twice as large as predicted; the vector meson contribution in the next-to-leading order (sometimes parametrized by an effective coupling constant a_v) is considered to be important to this decay [8].

One of the consequences of the unitarity corrections to $K^+ \rightarrow \pi^+ \gamma \gamma$ is a non-zero amplitude in the kinematic region close to the end point of $P = 227$ MeV/ c (the two-photon invariant mass $m_{\gamma\gamma} = 0$ MeV/ c^2), as shown in Fig. 1. The partial branching ratio $B(K^+ \rightarrow \pi^+ \gamma \gamma, P > 213 \text{ MeV}/c)$, corresponding to $m_{\gamma\gamma} < 108 \text{ MeV}/c^2$, is predicted to be $6.10^{+0.16}_{-0.12} \times 10^{-9}$ for $\hat{c} = 1.8 \pm 0.6$ including unitarity corrections and $0.49^{+0.23}_{-0.18} \times 10^{-9}$ for $\hat{c} = 1.6 \pm 0.6$ without the corrections. The former is one order of magnitude larger than the latter, and is within reach of E949. Observation of the decay at a partial branching ratio larger than predicted would indicate the contribution of vector-meson exchange or other new dynamics to $K^+ \rightarrow \pi^+ \gamma \gamma$. The kinematic region close to the end point in $K_L^0 \rightarrow \pi^0 \gamma \gamma$ is known to be crucial to understand the CP-conserving component to the $K_L^0 \rightarrow \pi^0 e^+ e^-$ decay through the $K_L^0 \rightarrow \pi^0 \gamma \gamma \rightarrow \pi^0 e^+ e^-$ amplitude, but experimental results on a_v in $K_L^0 \rightarrow \pi^0 \gamma \gamma$ [9,10] are inconsistent (see Ref. [11,12] for theoretical discussions).

E949 is primarily designed to measure the decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ [13]. The AGS delivered kaons of 710 MeV/ c to the experiment at a rate of 12.8×10^6 per 2.2-s spill. Kaons, detected and identified by Čerenkov, tracking, and energy-loss counters, were slowed by BeO and active degraders, and came to rest and decayed in a scintillating-fiber target. Fig. 2 shows a diagram of the apparatus. Measurements of charged decay products were made using the target, a central drift chamber, and a cylindrical range stack (RS) composed of 17

layers of 2-cm thick plastic scintillator with two embedded layers of tracking chambers. The pion from the $K^+ \rightarrow \pi^+ \gamma \gamma$ decay was identified by observation of the $\pi^+ \rightarrow \mu^+ \rightarrow e^+$ decay sequence in the RS using 500-MHz flash-ADC waveform digitizers [14]. Sets of thin trigger counters (“I” and “T” in Fig. 2) surrounding the drift chamber defined the fiducial region. (The T counters are regarded as the first layer of the RS.) Thin counters surrounding the RS identified (and suppressed) the muons from $K^+ \rightarrow \mu^+ \nu$ and $K^+ \rightarrow \mu^+ \nu \gamma$ decays whose range is longer than that of the signal. A hermetic calorimeter system surrounded the central region; the photons from $K^+ \rightarrow \pi^+ \gamma \gamma$ were detected in a lead/scintillator sandwich barrel detector (BV) surrounding the RS, while two endcap calorimeters and other detectors were used for detecting extra particles. A solenoid surrounding the BV provided a 1 T magnetic field along the beam line.

The beam and apparatus in E949, as well as the proton beam intensity from the AGS, were improved over those used in E787 [15] for the $K^+ \rightarrow \pi^+ \gamma \gamma$ study, which was performed in 1991. The kaon beam line [16], which incorporated two stages of particle separation, reduced pion contamination while increasing kaon acceptance. The target, central drift chamber, and RS tracking chambers were replaced by a new target consisting of 0.5-cm square fibers, a new low-mass drift chamber [17], and straw-tube chambers, respectively. One third of the RS scintillation counters were replaced to increase the light output. A new photon detector, the barrel veto liner (BVL), was installed to add 2.3 radiation lengths of lead/scintillator sandwich material to the BV. The endcaps were replaced by new fully-active detectors consisting of undoped-CsI crystals [18] with significantly increased light output, and both the target and endcaps were read out using 500-MHz CCD waveform digitizers [19] to

improve timing and double pulse resolution. Additional ancillary photon veto systems [20] and an LED flasher system to aid in the RS energy calibration were also introduced.

The new data were acquired in 2002, and the total exposure of kaons entering the target available for the $K^+ \rightarrow \pi^+ \gamma \gamma$ study in E949 was $N_K = 1.19 \times 10^{12}$. The trigger required a kaon decay at rest, followed by a π^+ track which came to rest in the RS and by coincident activity due to electromagnetic showers in both the BVL and BV, and no extra particles in the endcap or RS counters. The requirement of a sufficient time delay (≥ 1.5 ns) between the Čerenkov and I counter signals ensured that the kaons had decayed at rest in the target. The RS counter where the π^+ track came to rest, called the “stopping counter”, was required to be in the 16th or 17th layer in order to suppress π^+ tracks from the $K^+ \rightarrow \pi^+ \pi^0$ decay ($K_{\pi 2}$) with $P = 205$ MeV/ c . An improved trigger system [21] of E949, including a programmable trigger board, allowed more efficient running and reduced the online dead time. A total of 1.1×10^7 events met the trigger requirements.

The signature of $K^+ \rightarrow \pi^+ \gamma \gamma$ was a kaon decay at rest with a π^+ track in the RS in the kinematic region above the $K_{\pi 2}$ monochromatic peak and with photons reconstructed in the BVL and BV calorimeters. The momentum, the range (equivalent cm of plastic scintillator, R), and the kinetic energy (E) of the π^+ track were reconstructed with the target, drift chamber and RS information. The π^+ tracks in $213 \text{ MeV}/c < P < 234 \text{ MeV}/c$, $33.5 \text{ cm} < R < 41.3 \text{ cm}$, and $116 \text{ MeV} < E < 135 \text{ MeV}$ were accepted; the lower limits were 3.3, 2.3, and 2.6 standard deviations above the $K_{\pi 2}$ peak ($P = 205$ MeV/ c , $R = 30.4 \text{ cm}$, and $E = 108 \text{ MeV}$), respectively. Improvements in kinematic reconstruction permitted a larger search region ($P > 213 \text{ MeV}/c$)

compared to that in E787 ($P > 215 \text{ MeV}/c$) and removed the need for a constrained fit for consistency with $K^+ \rightarrow \pi^+ \gamma \gamma$ kinematics that was used in E787 [2]. The timing and energy (E_γ) of the photons were determined by grouping adjacent hit modules in the BV and BVL to identify isolated photon showers (“clusters”). The hit position in each module along the beam axis (z) was calculated from the end-to-end time and energy differences; the azimuthal angle (ϕ) of the hit position was determined by the segmentation of the modules. The location of the photon shower in z and ϕ was obtained by an energy-weighted average of the hit positions and was used, in conjunction with the kaon-decay vertex position in the target, to determine the polar and azimuthal angles of the photon to the π^+ track ($\theta_{\pi^+\gamma}$ and $\phi_{\pi^+\gamma}$). Since the opening angle between two photons from $K^+ \rightarrow \pi^+ \gamma \gamma$ gets smaller for the events with π^+ momentum close to the kinematic end point, the two photons of approximately half of the $K^+ \rightarrow \pi^+ \gamma \gamma$ decays are resolved as a single cluster, not two, in the BVL and BV. The events with either of one or two clusters in the final state were therefore accepted in the offline analysis. The higher-energy cluster should satisfy $50 \text{ MeV} < E_\gamma < 320 \text{ MeV}$, $\theta_{\pi^+\gamma} > 155^\circ$, and $\phi_{\pi^+\gamma} > 155^\circ$. The energy of the lower-energy cluster, if exists, should be at least 10 MeV.

The three background sources from kaon decays at rest can be classified as

“mismeasured” : $K_{\pi 2}$ decays with mismeasurements of the π^+ and the two photons,

“overlap” : $K_{\pi 2}$ decays with the softer photon overlapping the π^+ track in the RS (the kinetic energy of reconstructed π^+ tracks could be incorrectly measured due to additional energy deposited in the scintillators by the overlapping photon), and

“muon” : kaon decays with a muon misidentified as a π^+ and with photons in the final state (e.g. $K^+ \rightarrow \mu^+ \nu \gamma$, $K^+ \rightarrow \pi^0 \mu^+ \nu$ and $K_{\pi 2}$ with π^+ decay in flight).

The fourth background source is classified as

“DIF” : $K_{\pi 2}$ decay-in-flight before the kaon comes to rest in the target.

Beam-related backgrounds (e.g. multiple beam particles into the detector) were found to negligible.

These backgrounds were studied from the data by imposing offline selection criteria (“cuts”). The requirements on the π^+ momentum, range and kinetic energy provided large suppression of all of the backgrounds from kaon decays at rest. The cuts on the invariant mass of two photons (to reject events with $m_{\gamma\gamma} > 100 \text{ MeV}/c^2$), on extra activity (to reject events with activity not associated with the π^+ and the candidate signal photons⁷), and on the photon clusters (to reject events with two photons from a π^0 which hit the same or adjacent modules in BVL and BV and form a single cluster⁸) [22], referred to as “ γ selection cuts”, were imposed to suppress the mismeasured background. The dE/dx cuts in the RS (to reject events with a RS counter in which the

⁷ The extra activity was identified in the various subsystems, including the RS, as hits in the counters in coincidence with the π^+ track within a few ns and with energy above a threshold of typically 1 MeV. Only those events whose total photon-shower energy was deposited in the BVL and BV calorimeters were accepted.

⁸ Due to the kinematics of $K_{\pi 2}$ and subsequent $\pi^0 \rightarrow \gamma\gamma$ decays, the two photons must hit the modules at different z positions along the beam axis. An event was rejected if the maximum discrepancy among the z -position measurements in the modules of the cluster was larger than 113 cm.

measured energy was larger than expected from the reconstructed range in that counter) were imposed to suppress the overlap background [22]. The cuts on the relation between the range measured in the RS and the momentum measured in the drift chamber as well as the cuts on the $\pi^+ \rightarrow \mu^+ \rightarrow e^+$ decay sequence, recorded in the RS stopping counter, were imposed to suppress the muon background. The DIF background was suppressed by the cuts on the delay (≥ 2 ns) between the π^+ time and the K^+ time measured in the target and the cuts on the timing between the π^+ in the RS and the K^+ in the Čerenkov counter.

To study and measure these backgrounds, two independent sets of cuts were established for each background source. At least one of these cuts was inverted to enhance each background collected by the $K^+ \rightarrow \pi^+ \gamma \gamma$ trigger as well as to prevent candidate events from being examined before the background studies were completed. To avoid contamination from other background sources, all the cuts except for those being studied were imposed on the data. Possibilities of a correlation between the two sets of cuts or of a biased estimate of the effectiveness of the cuts were studied [23] in the same manner as the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ analysis of E787 [24] and E949 [13]. The level of signal acceptance as a function of cut severity and the comparison of the observed background levels near but outside the signal region with the predicted background in these regions were studied in the same manner. Table 1 summarizes the background levels measured with the final analysis cuts and the two sets of cuts for studying each background source. In total, 0.197 ± 0.070 background events were expected in the signal region.

The acceptance (A) and the single event sensitivity (SES) for $K^+ \rightarrow \pi^+ \gamma \gamma$ in the kinematic region $P > 213$ MeV/ c were derived from the acceptance

factors in Table 2 and the total kaon exposure $N_K = 1.19 \times 10^{12}$ times the K^+ -stopping efficiency, which is the fraction of kaons entering the target that came to rest (measured to be 0.754 ± 0.024 with the $K_{\pi 2}$ events collected by the $K^+ \rightarrow \pi^+ \gamma \gamma$ trigger). We obtained $A = (2.99 \pm 0.07) \times 10^{-4}$ and $SES = (3.72 \pm 0.14) \times 10^{-9}$ for $\hat{c} = 1.8$ including unitarity corrections and $A = (1.10 \pm 0.04) \times 10^{-4}$ and $SES = (1.01 \pm 0.05) \times 10^{-8}$ for $\hat{c} = 1.6$ without the corrections. The former sensitivity was below the predicted branching ratio of 6.10×10^{-9} , giving an expectation of 1.6 events. In order to verify that the sensitivity estimations were correct, a sample of $K^+ \rightarrow \mu^+ \nu$ decays accumulated by a calibration trigger was analyzed. The measured branching ratio of 0.628 ± 0.020 was consistent with the Particle Data Group value [7].

After imposing all analysis cuts, no events were observed in the signal region (Fig. 3). The group of 74 events around $R = 32$ cm and $E = 110$ MeV are due to the $K_{\pi 2}$ background. (“32 cm” and “110 MeV” are slightly larger than those expected for the $K_{\pi 2}$ decay, because the stopping-layer requirement in the trigger collected the $K_{\pi 2}$ events whose range and energy were measured to be larger in the RS.) Taking 2.24 events instead of zero according to the unified approach [25] with the background contribution of 0.197 events, we set a 90% C.L. upper limit on the partial branching ratio $B(K^+ \rightarrow \pi^+ \gamma \gamma, P > 213 \text{ MeV}/c)$ as 8.3×10^{-9} for $\hat{c} = 1.8$ including unitarity corrections and 2.3×10^{-8} for $\hat{c} = 1.6$ without the corrections. The systematic uncertainty was not taken into consideration in deriving the limits. For the purpose of comparison with the previous E787 results, a 90% C.L. upper limit for the total $K^+ \rightarrow \pi^+ \gamma \gamma$ branching ratio assuming the phase-space distribution was calculated; the present limit 6.0×10^{-8} is 8.3 times better than the same limit in E787 (5.0×10^{-7}).

The data described above were used to set a 90% C.L. upper limit on the branching ratio for $K^+ \rightarrow \pi^+ \gamma$ decay, which is forbidden by angular-momentum conservation and by gauge invariance but is allowed in noncommutative theories [26]. The signature of $K^+ \rightarrow \pi^+ \gamma$ was a two-body decay of a kaon at rest with a 227-MeV/ c π^+ track in the RS and a 227-MeV photon emitted directly opposite to it and observed as a single cluster in the BVL and BL calorimeters. The trigger, event reconstruction, and offline selection criteria in the study of $K^+ \rightarrow \pi^+ \gamma \gamma$ had been designed so that the same data were available to the search for $K^+ \rightarrow \pi^+ \gamma$. Since the background levels were already very small, the π^+ accepted region was not reduced for the $K^+ \rightarrow \pi^+ \gamma$ analysis. The previous limit from the E787 study was 3.6×10^{-7} [22], which was performed in 1996 and 1997 with a highly prescaled trigger with relaxed conditions resulting in the total exposure of kaons to be 6.7×10^8 . The new limit from E949, using the acceptance for $K^+ \rightarrow \pi^+ \gamma$ of $(1.08 \pm 0.02) \times 10^{-3}$, is 2.3×10^{-9} .

The results from this study cannot confirm nor rule out the unitarity corrections of ChPT, but the upper limits obtained are the tightest yet achieved on $K^+ \rightarrow \pi^+ \gamma \gamma$ and as well as on $K^+ \rightarrow \pi^+ \gamma$. The analysis, which was limited by the total exposure of kaons during the data collection in 2002, has shown that the experiment with kaon decays at rest is suitable to study $K^+ \rightarrow \pi^+ \gamma \gamma$ in the π^+ momentum region close to the end point. The possibility to observe the $K^+ \rightarrow \pi^+ \gamma \gamma$ decay in the kinematic region, if the ChPT including unitarity corrections is correct, gives further impetus for additional data collection.

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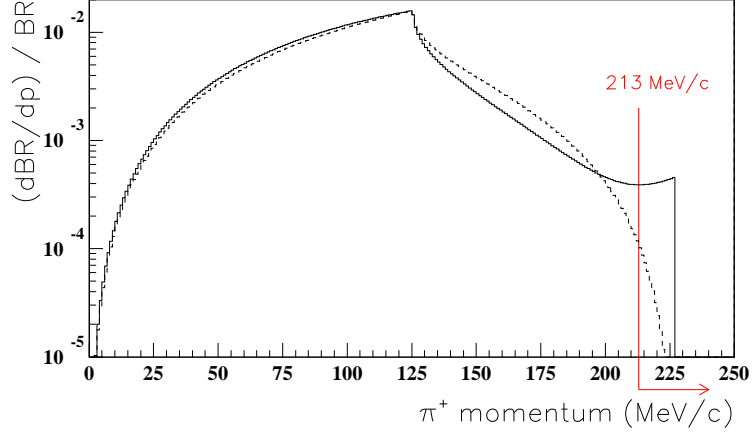


Fig. 1. Predictions for the π^+ momentum for $\hat{c} = 1.8$ including unitarity corrections (solid line) and for $\hat{c} = 1.6$ without the corrections (dashed line), respectively. The ratio of the partial branching ratio in the kinematic region $P > 213 \text{ MeV}/c$, indicated by the arrow, to the total branching ratio is 5.77×10^{-3} in the former and 0.515×10^{-3} in the latter.

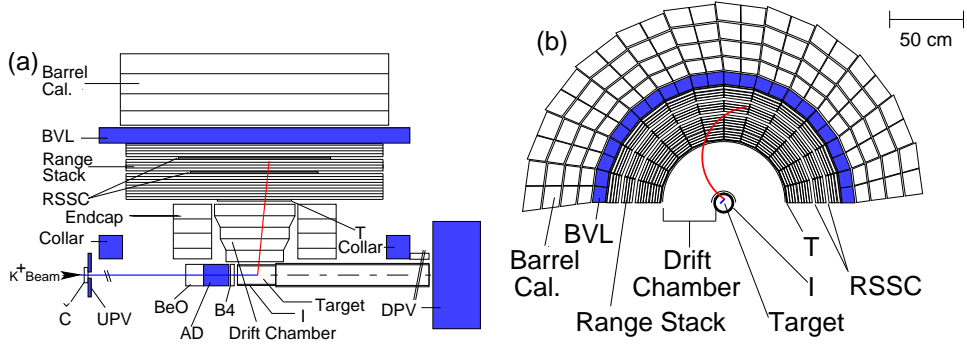


Fig. 2. Schematic side (a) and end (b) views of the upper half of the E949 detector. Č: Čerenkov counter; B4: energy-loss counters; I and T: inner and outer trigger scintillation counters; RSSC: RS straw-tube tracking chambers. New or upgraded subsystems for E949 (shaded) included the barrel veto liner (BVL), collar, upstream photon veto (UPV), active degrader (AD), and downstream photon veto (DPV). In addition, the outer trigger detectors and other trigger and electronics systems were improved in E949.

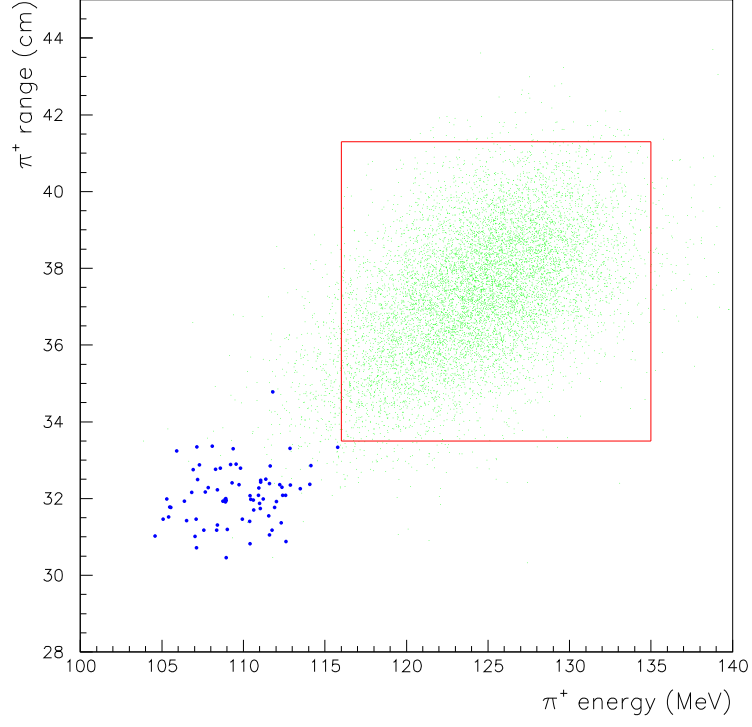


Fig. 3. Range vs. kinetic energy plot of the events with all analysis imposed. The box indicates the signal region for the $K^+ \rightarrow \pi^+ \gamma \gamma$ decay. The dark points represent the data. The simulated distribution of expected events from $K^+ \rightarrow \pi^+ \gamma \gamma$ for $\hat{c} = 1.8$ including unitarity corrections is indicated by the light dots.

Source	background level	two sets of cuts	
mismeasured	0.017 ± 0.006	P, R, E	γ selection
overlap	0.065 ± 0.065	P, R	dE/dx
muon	0.090 ± 0.020	P, R, E	$\pi^+ \rightarrow \mu^+ \rightarrow e^+$
		range-momentum	
DIF	0.025 ± 0.014	delay in the target	RS - Č timing

Table 1

Expected background levels in the signal region and the two sets of cuts for studying each background source. The total background was 0.197 ± 0.070 events.

Acceptance factors	UC	w/o UC	samples
Trigger	0.0666	0.0435	MC
Trigger-counter efficiency	0.936	0.936	K_B
charged-track reconstruction	0.996	0.996	$K_{\mu 2}$
π^+ fiducial cuts	0.984	0.938	MC
π^+ accepted region (P, R, E)	0.912	0.667	MC
π^+ stop without nuclear interaction	0.492	0.524	MC
or decay-in-flight			
dE/dx and kinematic cuts	0.537	0.537	$K_{\pi 2}, \pi_{scat}$
$\pi^+ \rightarrow \mu^+ \rightarrow e^+$ cuts	0.349	0.349	π_{scat}
γ reconstruction and fiducial cuts	0.530	0.492	MC, $K_{\pi 2}$
γ selection cuts	0.216	0.177	MC, $K_{\pi 2}$
Other cuts on beam and target	0.507	0.507	$K_{\mu 2}$
Total acceptance	2.99×10^{-4}	1.10×10^{-4}	

Table 2

Acceptance factors for the $K^+ \rightarrow \pi^+ \gamma \gamma$ decay in the kinematic region $P > 213$ MeV/ c , for $\hat{c} = 1.8$ including unitarity corrections (“UC”) and for $\hat{c} = 1.6$ without the corrections (“w/o UC”), and the samples used to determine them. “MC” in the rightmost column means the sample generated by Monte Carlo simulation. “ K_B ”, “ $K_{\mu 2}$ ”, “ $K_{\pi 2}$ ”, and “ π_{scat} ” mean the data samples of kaons entering the target, $K^+ \rightarrow \mu^+ \nu$ decays, $K_{\pi 2}$ decays, and scattered beam pions, respectively; these samples were accumulated by calibration triggers simultaneous to the collection of signal candidates.